

**FURTHER COMPUTATIONS OF THE CONSEQUENCES OF SETTING
THE ANNUAL KRILL CATCH LIMIT TO A FIXED FRACTION
OF THE ESTIMATE OF KRILL BIOMASS FROM A SURVEY**

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Abstract

Butterworth *et al.* (1992) put forward an extension of the harvesting model of Beddington and Cooke (1983) to relate potential krill yield to a pre-exploitation survey estimate of krill biomass. In this paper, the approach is extended further so as to incorporate most of the amendments specified by the Third and Fourth Meetings of the Working Group on Krill (WG-Krill). The most important of these extensions is integration over the ranges of uncertainty for a number of model parameters. Results are provided for the probability of spawning biomass falling below various fractions of its median pre-exploitation level (K^{sp}), as a function of the fraction of the biomass estimate which is set as the catch for a 20-year period. Three possible fishing seasons are considered. The model extensions requested by the Third Meeting make little difference to the results of Butterworth *et al.* (1992). Winter fishing is marginally preferable to a summer harvest. However, the imposition of an upper bound of 1.5 yr^{-1} on the effective annual fishing mortality, as specified by the Fourth Meeting, results in marked reductions in the probabilities of krill spawning biomass falling below specified fractions of K^{sp} .

Résumé

Dans Butterworth *et al.* (1992), les auteurs présentent un complément au modèle d'exploitation de Beddington et Cooke (1983) établissant la relation entre le rendement potentiel du krill et l'estimation de la biomasse du krill provenant d'une campagne d'évaluation antérieure à l'exploitation. Cette approche y est développée pour incorporer la plupart des amendements spécifiés par les troisième et quatrième réunions du Groupe de travail sur le krill (WG-Krill). L'intégration des intervalles d'incertitude d'un certain nombre de paramètres du modèle forme le développement le plus important. Les auteurs fournissent les résultats du calcul de probabilité que la biomasse reproductrice tombe au-dessous de diverses proportions de son seuil médian avant l'exploitation (K^{sp}), représentant la proportion de la biomasse estimée sur laquelle est fixée la capture pour une période de 20 ans. Trois saisons de pêche potentielles sont examinées. Les développements du modèle requis à la troisième réunion ne sont guère différents des résultats de Butterworth *et al.* (1992). La pêche d'hiver semble légèrement préférable à celle d'été. Toutefois, la limite de $1,5 \text{ an}^{-1}$ imposée pour la mortalité par pêche annuelle réelle, spécifiée à la quatrième réunion, réduit nettement la probabilité que la biomasse reproductrice du krill chute au-dessous des proportions fixées de K^{sp} .

Резюме

В работе Баттеруорт и др. (Butterworth *et al.*, 1992) предлагается дополнить промысловую модель Беддингтона и Кука (Beddington and Cooke, 1983) с целью соотнесения потенциального вылова криля с доэксплуатационной съёмочной оценкой биомассы криля. В настоящей работе этот подход дополнен с целью внедрения большинства поправок, определенных на Третьем и Четвертом совещаниях Рабочей группы по крилю (WG-Krill). Наиболее важным из этих

дополнений является интеграция по рядам неопределенности для некоторых параметров модели. Приводятся величины вероятности того, что нерестующая биомасса упадет ниже различных долей своего медианного доэксплуатационного уровня (K^{sp}), как функция доли оценки биомассы, которая установлена как вылов за двадцатилетний период. Рассматриваются три альтернативных промысловых сезона. Дополнения к модели, запрошенные на Третьем совещании, существенно не влияют на результаты работы Баттеруорт и др. (Butterworth *et al.*, 1992). Зимний промысел немного предпочтительнее летнего. Тем не менее, наложение верхнего предела в $1,5 \text{ года}^{-1}$ на фактическую ежегодную промысловую смертность, как определено на Четвертом совещании, приводит к существенному уменьшению вероятности спада нерестующей биомассы криля ниже конкретных долей K^{sp} .

Resumen

Butterworth *et al.* (1992) presentaron una modificación del modelo de explotación elaborado por Beddington y Cooke (1983) para relacionar el rendimiento potencial de krill con una estimación de biomasa de krill derivada de una prospección previa a la explotación. En este documento este enfoque es ampliado para incorporar la mayor parte de las modificaciones indicadas en la Tercera y Cuarta reunión del Grupo de Trabajo del Krill (WG-Krill). La modificación más importante es la integración de los rangos de incertidumbre para diferentes parámetros del modelo. Se presentan los resultados para la hipótesis de que la biomasa reproductora descienda a distintos porcentajes de su nivel mediano previo a la explotación (K^{sp}), como una función de la fracción de biomasa estimada, la cual está fijada para dar la captura en un período de 20 años. Se han considerado tres posibles períodos de pesca. Los resultados obtenidos luego de efectuar las modificaciones solicitadas en la Tercera reunión apenas difieren de los resultados de Butterworth *et al.* (1992). La pesca efectuada en invierno es ligeramente favorable a la pesca realizada en el período estival. Sin embargo, la fijación de un límite superior de 1.5 año^{-1} en la mortalidad anual efectiva causada por la pesca, según lo especificado en la Cuarta reunión, reduce considerablemente la probabilidad de que la biomasa reproductora de krill descienda por debajo de proporciones determinadas de K^{sp} .

Keywords: krill, catch limit, biomass survey, CCAMLR

INTRODUCTION

Butterworth *et al.* (1992) carried out calculations for the parameter λ in the yield equation $Y = \lambda MB_0$ for a krill fishery. Their work was an extension of the harvesting model of Beddington and Cooke (1983), to allow for the fact that krill growth varies systematically through the year. Specifically, they assumed that krill growth and fishing were coincident over a three-month period of the year. Their calculations were based on the criterion that there is a 10% probability that the krill spawning biomass drops below a fraction $D_{crit} = 0.2$ of its median pre-exploitation level (K^{sp}) over a 20-year harvesting period.

The results which they obtained were particularly sensitive to values assumed for natural mortality (M) and recruitment variability (σ_R), as well as to the values for age-at-first-capture and maturity. The Third Meeting of WG-Krill subsequently requested that further calculations be carried out which would integrate

results over the ranges of uncertainty for these parameters, and thereby provide a crisper summary of the implications of the analysis (SC-CAMLR, 1991). In addition, refinements to the model were suggested to better reflect the real situation, and results for choices other than 0.2 for D_{crit} were requested.

This paper reports the results of these further calculations, which have been extended to incorporate some of the further amendments requested by the Fourth Meeting of WG-Krill (SC-CAMLR, 1992).

MODEL MODIFICATIONS

The model used by Butterworth *et al.* (1992) assumed knife-edge maturity and selectivity, and that the three-month growth and fishing periods for krill coincided. The choice of parameter values for their central 'base case' was as follows:

$$\begin{aligned}
M &= 0.6 \text{ yr}^{-1} \text{ (natural mortality)} \\
\sigma_R &= 0.4 \text{ (recruitment variability)} \\
a_r &= 3 \text{ yr (knife-edge age-at-recruitment)} \\
a_m &= 3 \text{ yr (knife-edge age-at-maturity)} \\
a+ &= 1+ \text{ yr (age-range corresponding to} \\
&\quad \text{biomass survey estimate)} \\
\sigma_s &= 0.3 \text{ (CV - approximately - of biomass} \\
&\quad \text{survey estimate)}.
\end{aligned}$$

Spawning biomass was taken to be the biomass of mature animals at the start of the growth period. This model will be referred to as 'model 1' hereafter, and results from it will be provided for comparative purposes.

For 'model 2' - the model with modifications specified by the Third Meeting of WG-Krill (SC-CAMLR, 1991) - the values of $a+$ and σ_s are retained as above. However, those for the other four parameters are drawn at random from pre-specified distributions for each simulation of a 20-year period of harvesting. For M and σ_R , these distributions are:

$$\left. \begin{aligned}
M &\text{ from } U [0.4, 1.0] \text{ yr}^{-1} \\
\sigma_R &\text{ from } U [0.4, 0.6]
\end{aligned} \right\} \quad (1)$$

where $U [a, b]$ is a uniform distribution over the range $[a, b]$.

Instead of knife-edge forms for the selectivity and maturity-at-age functions, these are modelled by a different shape as a function of length (ℓ). Thus selectivity:

$$S_\ell = \begin{cases} 0 & \text{for } \ell < \ell_1 = \ell_r^{50} - w_r / 2 \\ (\ell - \ell_1) / (\ell_2 - \ell_1) & \text{for } \ell_1 \leq \ell \leq \ell_2 \\ 1 & \text{for } \ell > \ell_2 = \ell_r^{50} + w_r / 2 \end{cases} \quad (2)$$

and for the proportion mature-at-length:

$$P_\ell = \begin{cases} 0 & \text{for } \ell < \ell_1 = \ell_m^{50} - w_m / 2 \\ (\ell - \ell_1) / (\ell_2 - \ell_1) & \text{for } \ell_1 \leq \ell \leq \ell_2 \\ 1 & \text{for } \ell > \ell_2 = \ell_m^{50} + w_m / 2 \end{cases} \quad (3)$$

where ℓ_r^{50} from $U [38, 42]$ mm with $w_r = 10$ mm,
and
 ℓ_m^{50} from $U [34, 40]$ mm with $w_m = 12$ mm.

These lengths can be converted into ages using the growth curve of Rosenberg *et al.* (1986):

$$\ell_{a,t} = \begin{cases} \ell_\infty [1 - e^{-\beta(a+4t)}] & 0 \leq t \leq 3 / 12 \\ \ell_\infty [1 - e^{-\beta(a+1)}] & 3 / 12 \leq t \leq 1 \end{cases} \quad (4)$$

where $\ell_\infty = 60$ mm
 $\beta = 0.45 \text{ yr}^{-1}$
 a is the age of the krill in years, and
 t measures time after the 'start' of the year.

As the three-month growth period corresponds to November-January, the 'start' of the year ($t = 0$) is 1 November.

The spawning biomass for model 2 is defined as the average biomass of mature krill over the December-March period ($t = 1/12$ to $5/12$), while the pre-exploitation biomass survey is assumed to take place in the middle of the growth season as for model 1 (i.e., in mid-December: $t = 1.5/12$).

As M now varies between simulations, it is no longer appropriate to express results in terms of λ where $Y = \lambda MB_0$. Instead, results are now given for γ , where:

$$Y = \gamma B_0 \quad (5)$$

i.e. $\gamma (= \lambda M)$ is the multiple of the biomass survey estimate which specifies the annual catch to be taken.

Since surveys may not cover the complete range of the stock, results were also requested for situations where the exact extent of the resultant negative bias of the survey estimate is unknown:

$$B_0 = f B_{\text{survey}} \quad (6)$$

where f indicates the multiple of the survey result required for an unbiased estimate of B_0 . The distribution to be used for f is:

$$f \text{ from } U [1.0, 4.0] \quad (7)$$

For such situations, the results are quoted in terms of δ , where:

$$Y = \delta B_{\text{survey}} = \lambda M f B_{\text{survey}} \quad (8)$$

i.e., $\delta = \lambda M f$.

Finally, results were requested for three different possible fishing seasons:

- (a) three months: December to February ($t = 1/12$ to $4/12$)
- (b) six months: April to September ($t = 5/12$ to $11/12$), and
- (c) uniform intensity throughout the year ($t = 0$ to 1).

The Fourth Meeting of WG-Krill (SC-CAMLR, 1992) requested further refinement of this model. Two of their specific requests have been incorporated here. First, a krill stock-recruit relationship is introduced: once krill spawning biomass drops below 20% of its median level in the absence of exploitation, median recruitment decreases proportionately to spawning biomass. Second, to place some realistic limit on the proportion of the recruitment which could be harvested in any year, an upper bound of 1.5 yr^{-1} is placed on the effective annual fishing mortality for fully selected age classes; this means that the complete fixed catch specified will not be taken in every year during the harvesting period.

Full details of the algebra underlying the requisite calculations, and the computational procedure, are given in the appendix.

RESULTS AND DISCUSSION

SC-CAMLR (1992) requested that statistics be provided for a 10- as well as a 20-year period of harvesting. Calculations revealed very little difference between such results, except for the modifications to probabilities of dropping below a critical biomass level which are to be expected from changing this period. Accordingly, in the interests of brevity, only the results for the 20-year period are reported here.

Results for the probability of abundance dropping below a critical value (D_{crit}) of B^{sp}/K^{sp} during this 20-year period of harvesting are shown primarily in graphical form, with this probability plotted against γ (or δ). Thus, Figure 1 shows such plots for values of D_{crit} from 0.1 to 0.6 for model 1, and for model 2 for the three different associated fishing seasons specified. Figure 2 shows the same plots enlarged for small values of γ and probability, while Figure 3 compares the curves for $D_{crit} = 0.2$ for different models and harvesting seasons. The values of γ

which correspond to a probability of 10% are shown in Table 1(a) for different values of D_{crit} .

To give an impression of the 'target' spawning biomass levels corresponding to different γ values, statistics of the distribution of B^{sp} (year 31)/ K^{sp} ($= B_{fin}/K^{sp}$) have been plotted against γ . Thus Figure 4 shows the median, 5% and 95% distribution points of this variable. Figure 5 compares the plots of the medians for different models and fishing seasons; Table 1(b) also provides such comparative results.

These figures and tables show that there is little qualitative difference in the results for models 1 and 2. Quantitative comparisons are most easily made by considering Figures 3(i) and 5(i). Probabilities for abundance falling below $D_{crit} = 0.2$ are somewhat higher for model 2; for the latter, summer fishing (December to February) leads to the highest probabilities for a given γ , and winter fishing (April to September) to the lowest, while the results for median B_{fin}/K^{sp} for model 2 show similar trends, with summer fishing resulting in the lowest values. However, the actual values differ very little from each other, or from those for model 1. The reason for the marginal preference for winter fishing is that, when the onset of fishing is delayed, gains from the increase in mass of individual krill over the growth period slightly exceed losses resulting from natural mortality (note that the growth model used for krill - see equations (A.4) and (A.5) - excludes the possibility that krill shrink during the winter).

Figure 6, together with Tables 1(b) and 2(b), provides a comparison of the results for the situation where the survey covers only a portion of the stock in question. Results are shown for the December-February and April-September harvesting seasons only; the results for fishing over the whole year are intermediate. As would be expected, larger values of δ than of γ are possible for the same risk of depletion.

Figure 7 shows trends in the average P/B (production/biomass) ratio over the 20-year harvesting period as γ is increased. The ratio increases, essentially because the population dynamics model assumed maintains the same value of median recruitment (a major contributor to annual production) as biomass drops with the increased fishing intensity. Results for model 2 for the three different fishing seasons scarcely differ, but are somewhat larger than those for model 1.

Table 1: Values of γ and δ for which the probability that the depletion ($D = B^{sp}/K^{sp}$) falls below D_{crit} over a 20-year period of harvesting is equal to 10% for the various models and harvesting strategies.

(a) γ (i.e., $Y = \gamma B_{obs}$)

Model and Harvesting Strategy		D_{crit}					
		0.1	0.2	0.3	0.4	0.5	0.6
1	$M = 0.6 \sigma_R = 0.4$	> 1	0.251	0.106	0.081	0.054	0.018
2(a)	December-February	0.254	0.136	0.106	0.080	0.049	0.008
2(b)	April-September	> 1	0.202	0.119	0.089	0.055	0.010
2(c)	Whole Year	> 1	0.165	0.110	0.083	0.051	0.009

(b) δ (i.e., $Y = \delta B_{survey}$)

Model and Harvesting Strategy		D_{crit}					
		0.1	0.2	0.3	0.4	0.5	0.6
2(a)	December-February	> 1	0.283	0.211	0.161	0.106	0.018
2(b)	April-September	> 1	0.489	0.244	0.177	0.117	0.021
2(c)	Whole Year	> 1	0.350	0.221	0.166	0.110	0.019

Table 2: Median values of the distributions of depletion at the end of the 20-year period of harvesting (B_{fin}/K^{sp}) for the various models and harvesting strategies considered.

(a) γ (i.e., $Y = \gamma B_{obs}$)

Model and Harvesting Strategy		γ						
		0.00	0.05	0.10	0.15	0.20	0.30	0.40
1	$M = 0.6 \sigma_R = 0.4$	1.000	0.882	0.745	0.609	0.507	0.434	0.415
2(a)	December-February	1.000	0.886	0.768	0.645	0.511	0.328	0.279
2(b)	April-September	1.000	0.896	0.787	0.673	0.573	0.467	0.434
2(c)	Whole Year	1.000	0.890	0.775	0.656	0.539	0.407	0.372

(b) δ (i.e., $Y = \delta B_{survey}$)

Model and Harvesting Strategy		δ						
		0.00	0.05	0.10	0.15	0.20	0.30	0.40
2(a)	December-February	1.000	0.948	0.893	0.838	0.788	0.692	0.591
2(b)	April-September	1.000	0.953	0.903	0.852	0.807	0.718	0.637
2(c)	Whole Year	1.000	0.950	0.897	0.843	0.794	0.702	0.612

Figure 8 shows the effect of imposing the constraint that the effective annual fishing mortality, F_y , has an upper bound (F_m) of 1.5 yr^{-1} . Figure 8(i) reveals that the probabilities of spawning biomass dropping below $D_{crit} = 0.2$ increase markedly when this constraint is weakened to $F_m = 5 \text{ yr}^{-1}$. Clearly, therefore, the asymptotic behaviour (at large γ) of the results in the preceding figures is a result of this constraint frequently coming into play when biomass is low and γ is large. This prevents the full intended catch being taken in the year concerned. This different behaviour is more marked in Figure 8(ii), which shows results for median B_{fin}/K^{sp} . For model 1, the asymptotic behaviour for large γ is still evident. The reason for the model behaving in this way is

that the knife-edge formulations for age-at-first-capture and maturity, together with the fact that B^{sp} in this model is measured at the 'start' of the year, mean that krill reaching three years of age always 'spawn' before fishing can have any impact on the abundance of that cohort; thus increased fishing intensity cannot reduce median B^{sp} below a certain level. In contrast, in model 2, the selectivity function allows for the capture of immature krill, so that increased fishing pressure can reduce median B^{sp} to a greater extent. Figure 8(ii) indicates that when the maximum possible fishing mortality is large ($F_m = 5 \text{ yr}^{-1}$), this reduction extends to zero; however, the $F_m = 1.5 \text{ yr}^{-1}$ constraint keeps catches of immature krill sufficiently low for median B^{sp} to tend to a non-zero limit for large γ .

This same effect is evident in Figure 9, which shows the consequences (under model 2 with summer fishing) of a lower length at 50% vulnerability and wider spread of the selectivity function (i.e., lower ℓ_r^{50} and higher w_r - see equation (2)). Catches then include a greater proportion of immature krill, with the result that probabilities of dropping below $D_{crit} = 0.2$ increase, and median B^{sp}/K^{sp} values drop (particularly for higher γ values).

CONCLUDING COMMENTS

Results requested by the Third Meeting of WG-Krill, and many of those requested by the following Fourth Meeting, have been provided. Aspects of the requests still to be investigated are: correlations between M , σ_R and the krill growth rate; age dependence of M ; sex differentiation; and censoring of the lower tail of the recruitment distribution.

Broadly, the extensions to the Butterworth *et al.* (1992) model specified by the Third Meeting make little difference to results, with probabilities of abundance falling below certain critical levels increasing only slightly. The Fourth Meeting's requested imposition of an upper bound of 1.5 yr^{-1} on the effective annual fishing mortality does, however, have a considerable effect, resulting in marked reductions in these probabilities.

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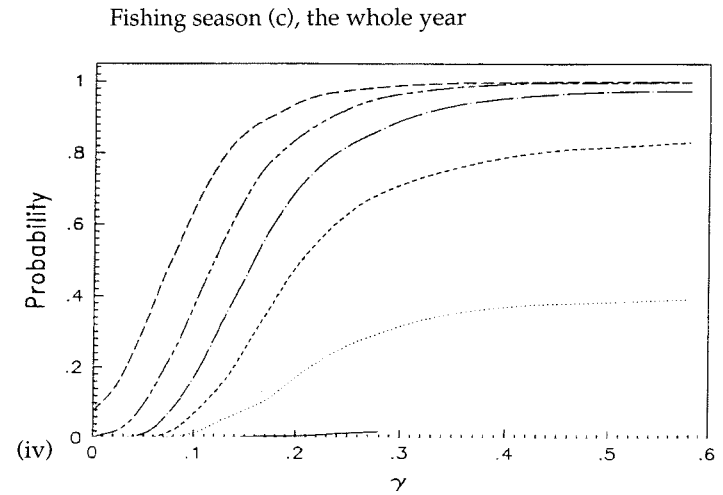
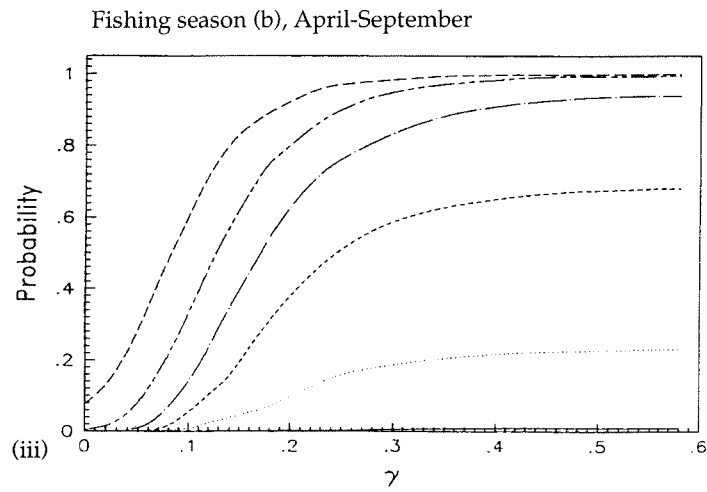
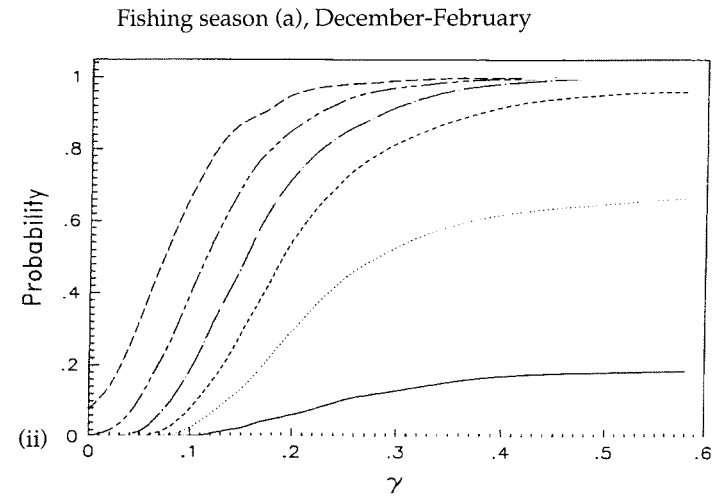
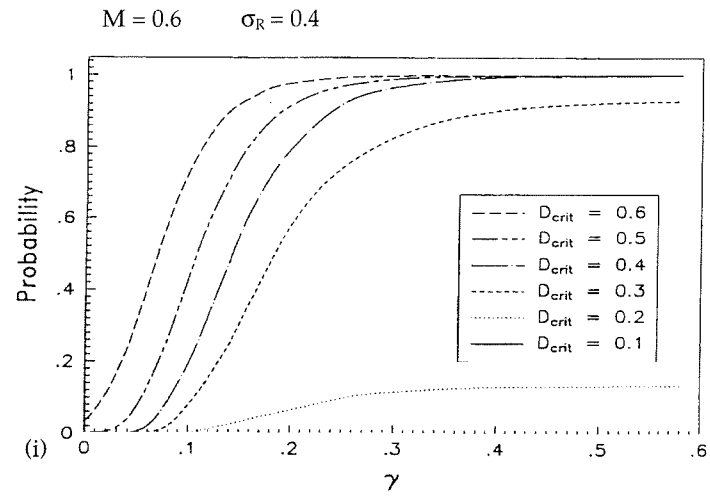


Figure 1: The probability that depletion ($D = B^{sp}/K^{sp}$) falls below the indicated value D_{crit} as a function of γ . Results shown in (i) are for model 1 of Butterworth *et al.* (1992) for $M = 0.6 \text{ yr}^{-1}$ and $\sigma_R = 0.4$. Those illustrated in the following plots are for model 2 of this paper with harvesting over (ii) December-February, fishing season (a); (iii) April-September, fishing season (b); and (iv) the whole year, fishing season (c).

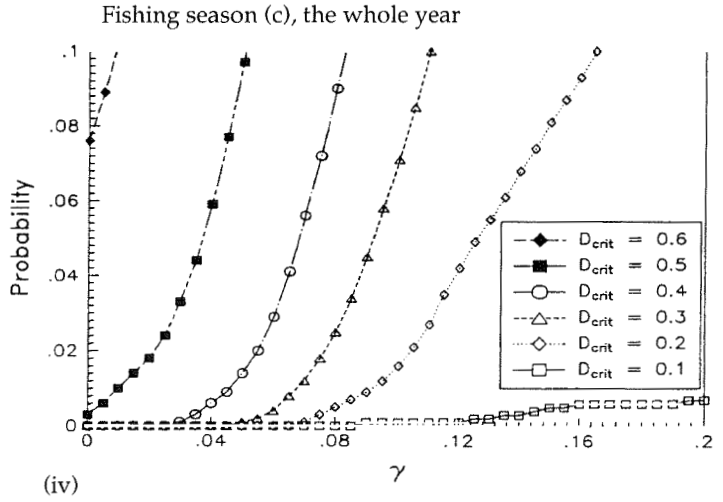
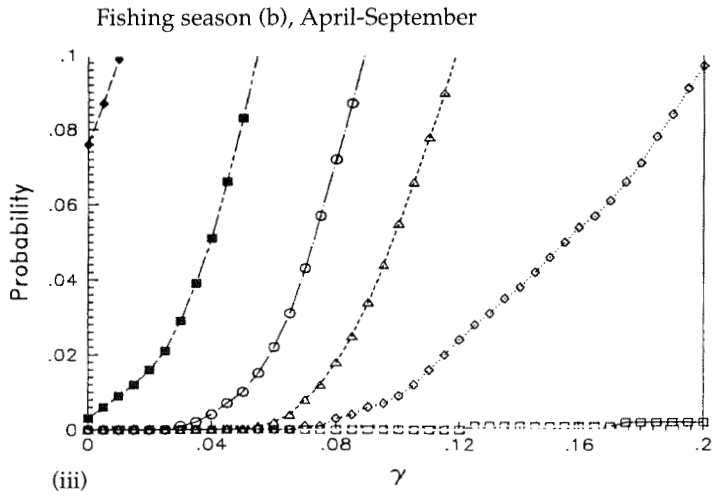
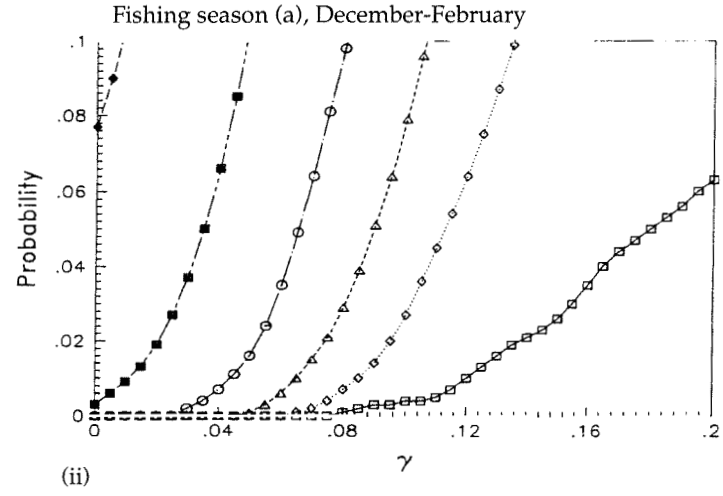
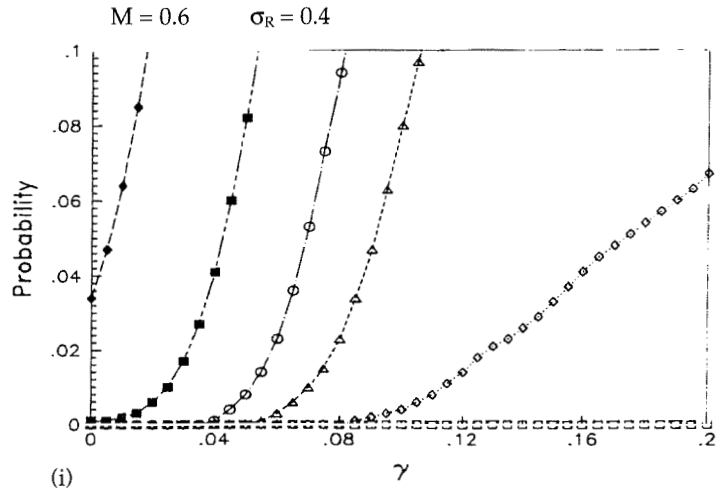


Figure 2: Figure 1 on an enlarged scale to show results for smaller values of γ and probability more clearly.

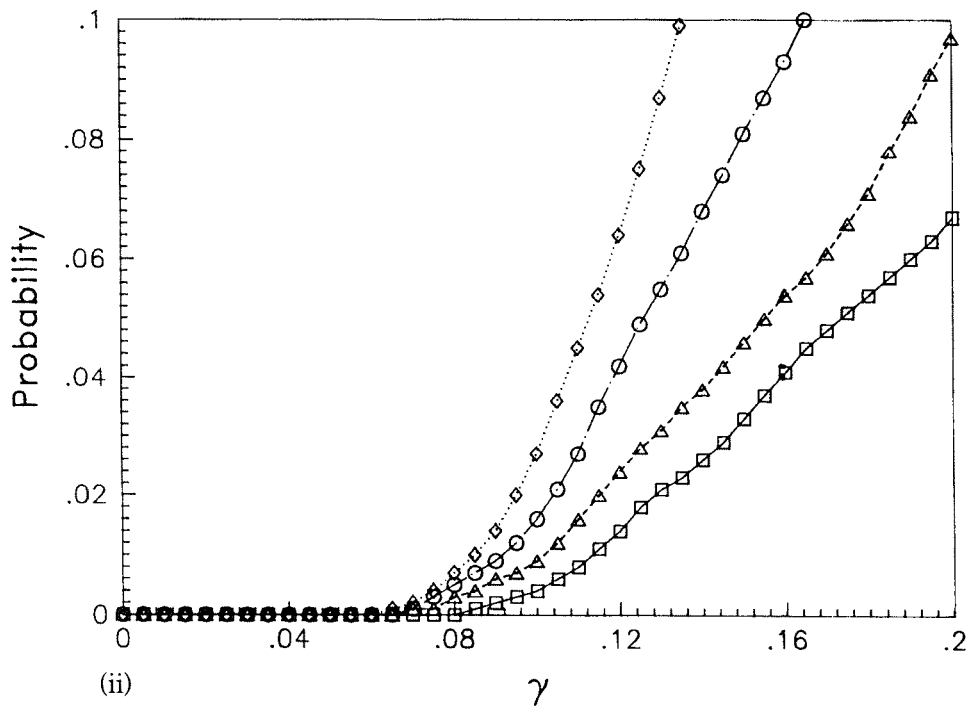
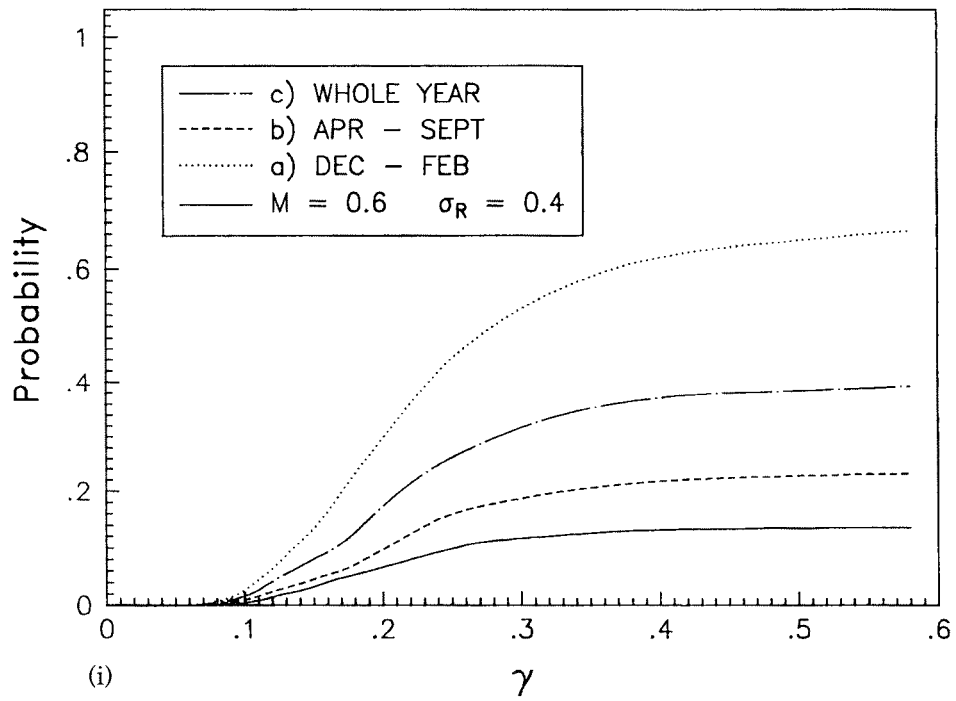


Figure 3: Plots of probability against γ for $D_{crit} = 0.2$ for model 1 and the three harvesting strategies associated with model 2. Plot (ii) is an enlarged version for smaller γ and lower probability values.

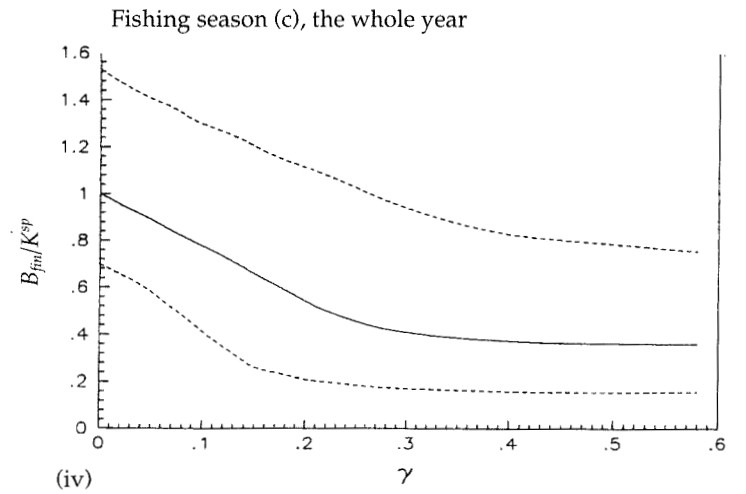
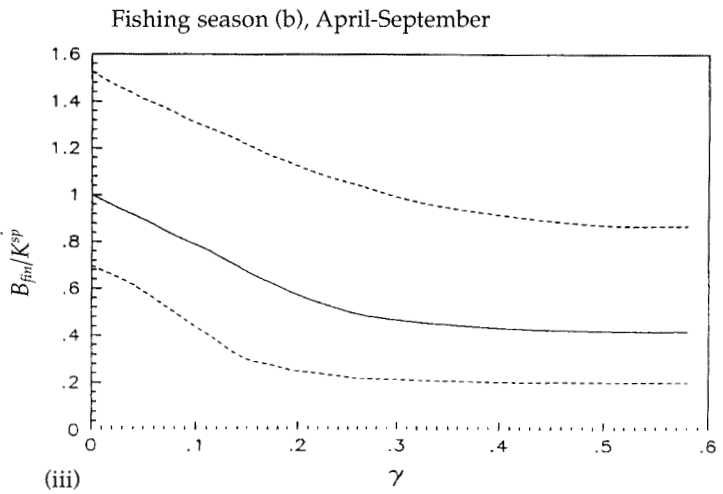
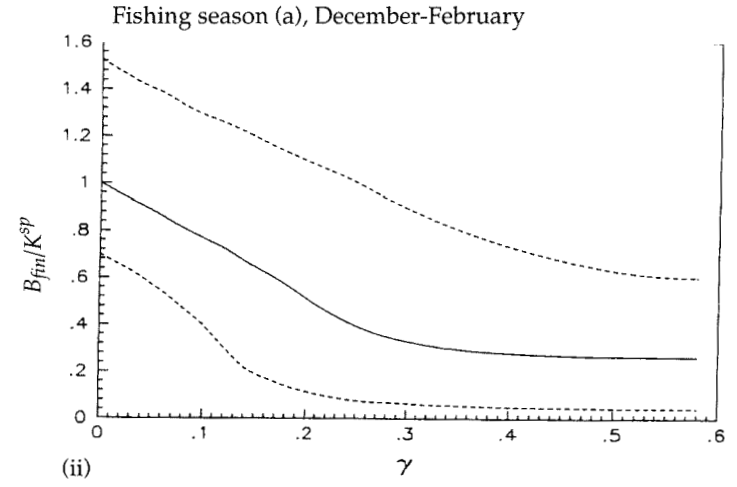
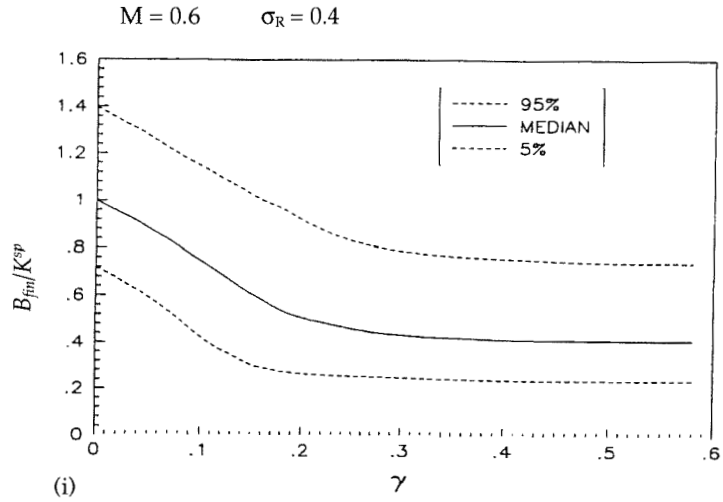


Figure 4: Distributions of B^{sp}/K^{sp} at the end of the harvesting period (B_{fin}/K^{sp}) plotted against γ . The median, 5% and 95% distribution points are shown. Plots (i) to (iv) are for the same scenarios as detailed in the Figure 1 caption.

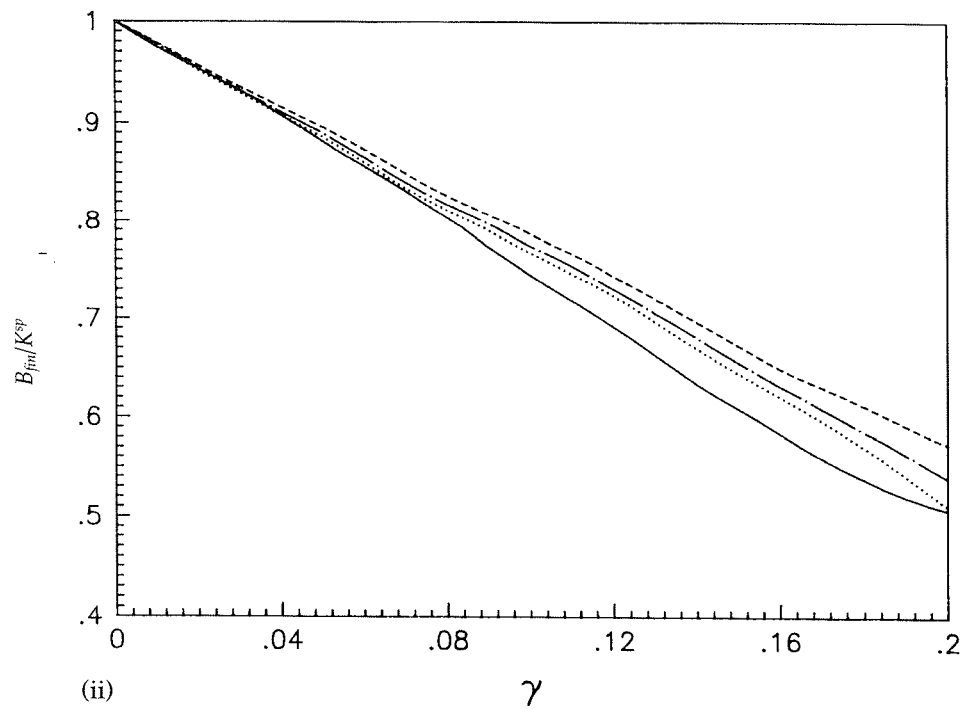
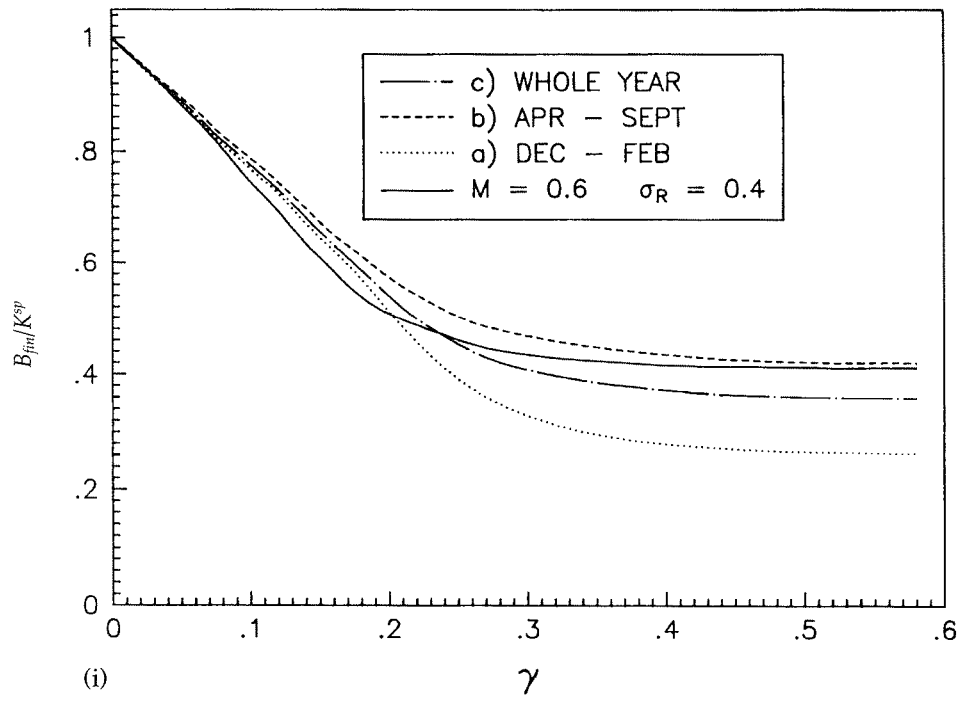


Figure 5: Comparison of medians of B_{fin}/K^{sp} distributions for model 1 and the three harvesting strategies associated with model 2. Plot (ii) is an enlarged version for smaller γ values.

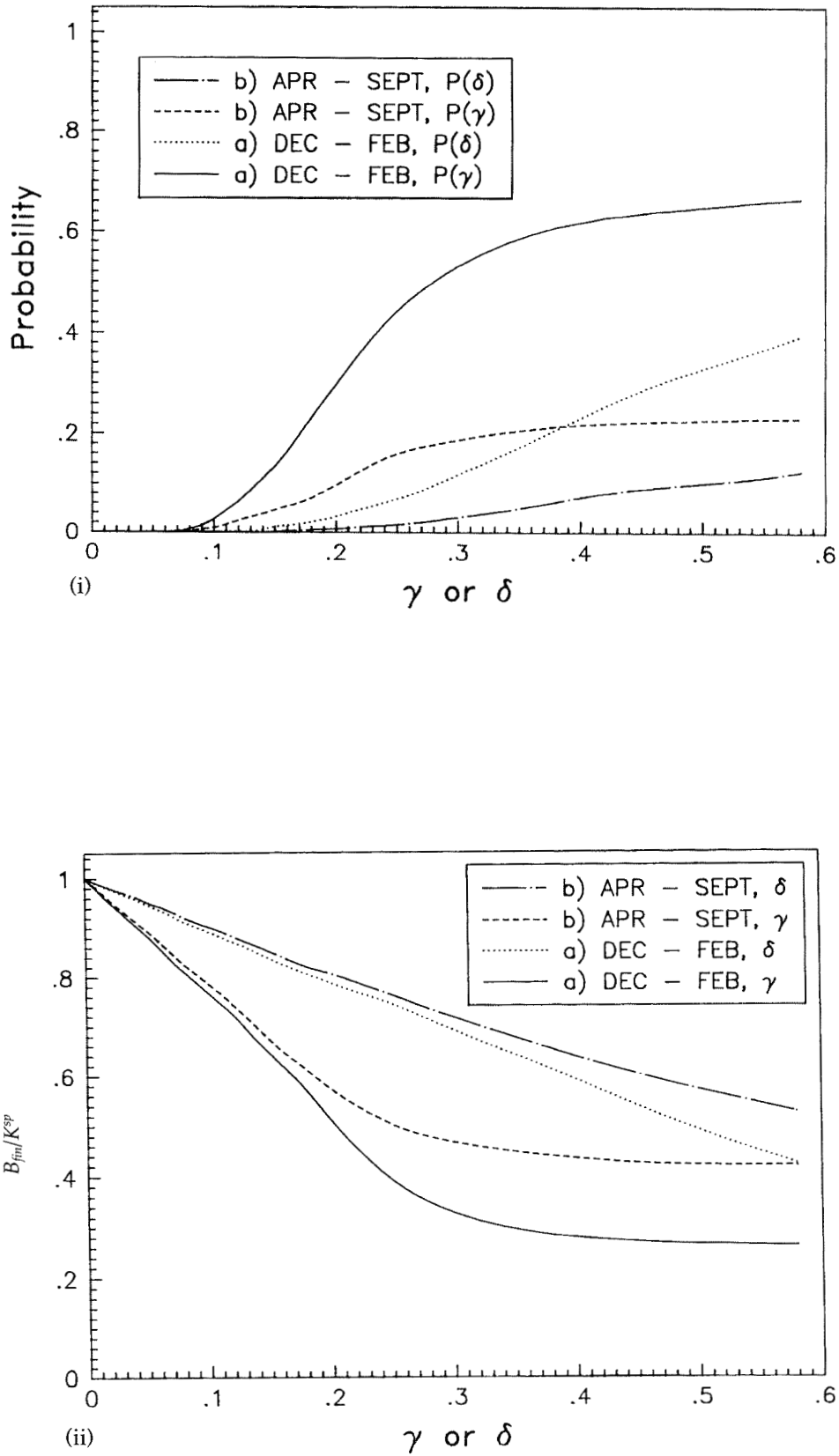


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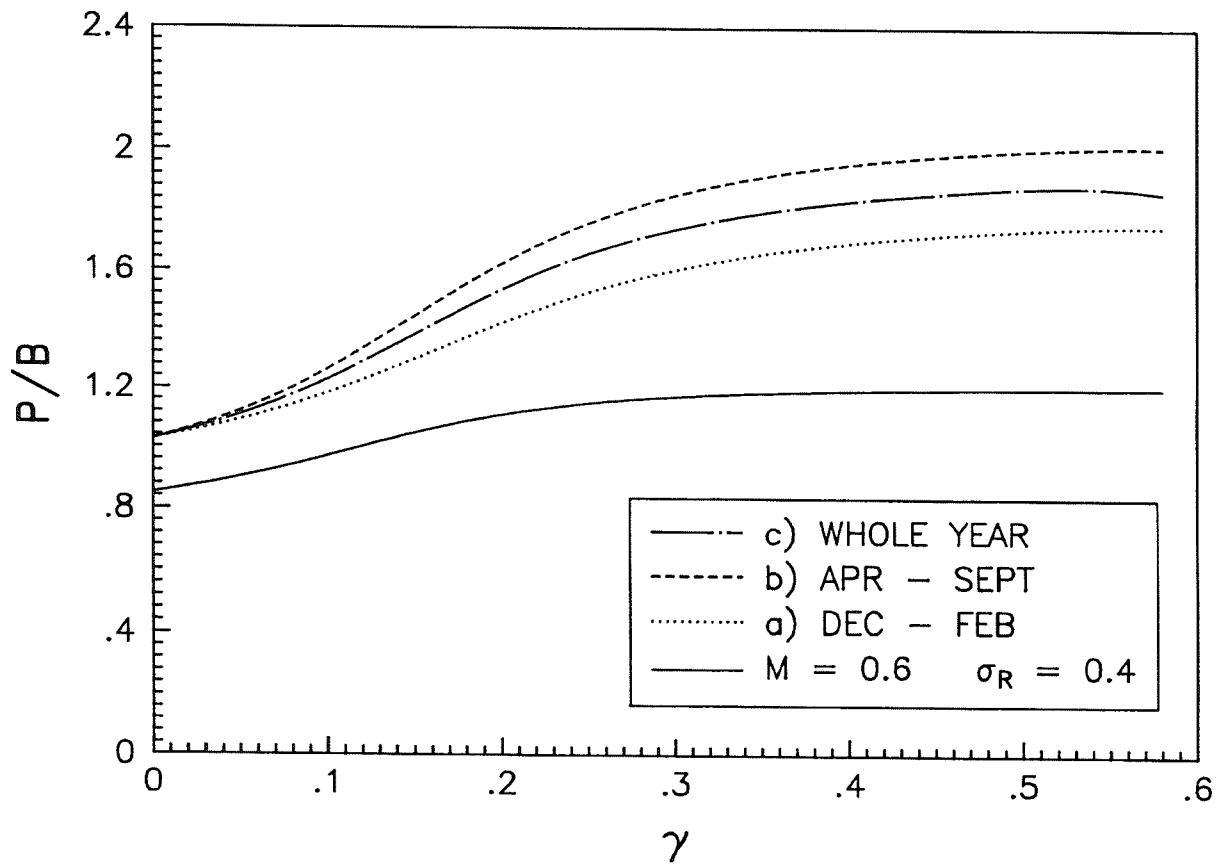


Figure 7: Comparison of plots of the average P/B (production/biomass) ratio over a 20-year harvesting period against γ for the four scenarios detailed in the Figure 1 caption.

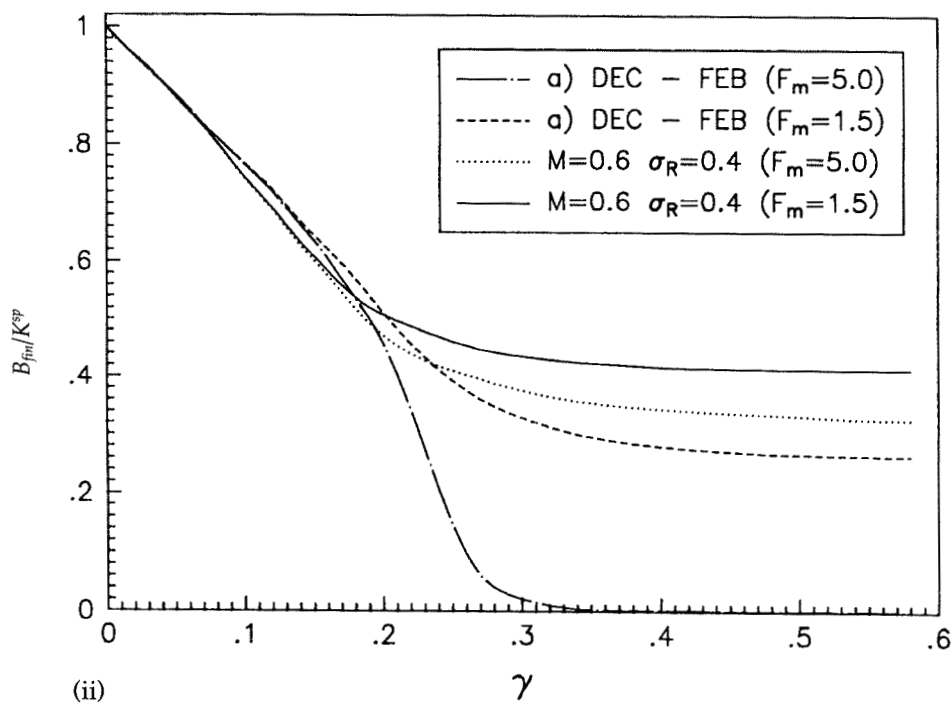
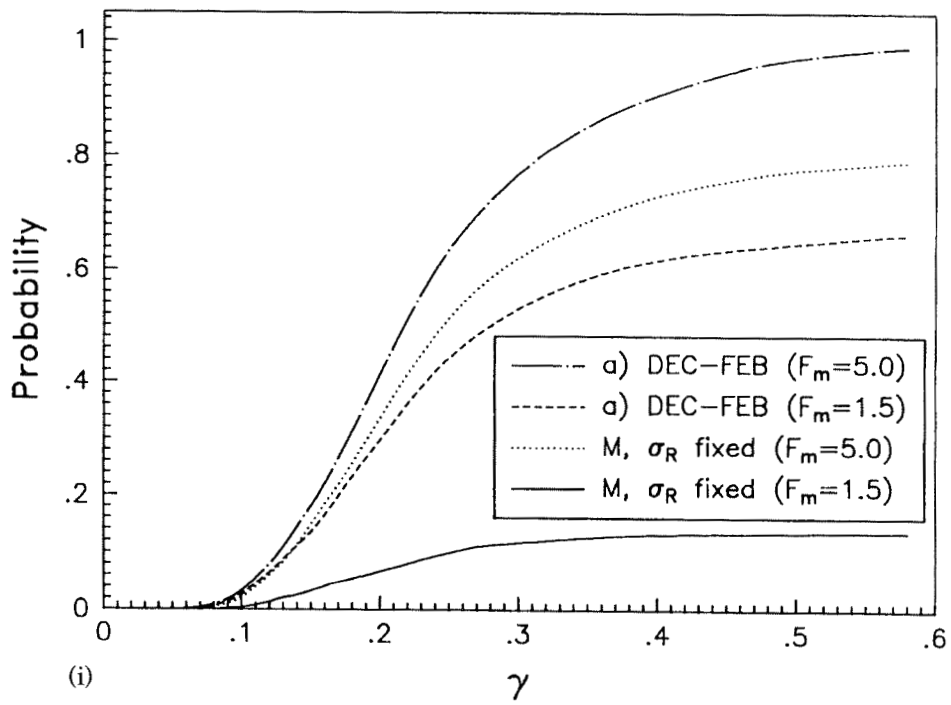


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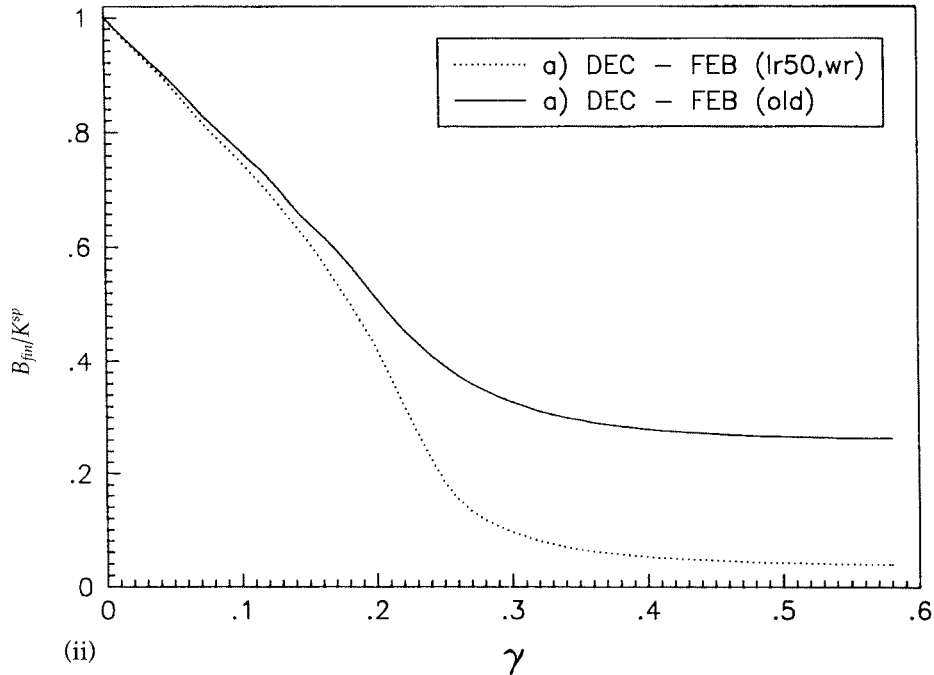
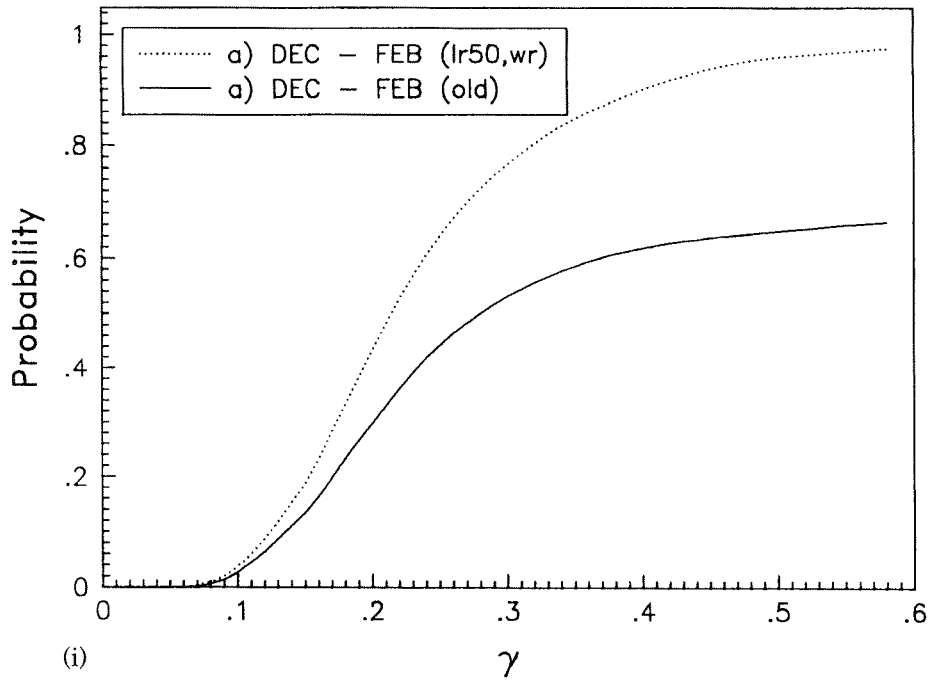


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MATHEMATICAL DETAILS OF MODELS AND CALCULATION PROCEDURES

1. KRILL DYNAMICS MODEL

For model 1 (Butterworth *et al.*, 1991) the numbers of krill of age a years at the start of 'year' y ($N_{y,a}$) are governed by the equations:

$$N_{y+1,a+1} = \begin{cases} N_{y,a} e^{-M} & 0 \leq a < a_r \\ N_{y,a} e^{-(M+F_y)} & a_r \leq a \leq 7 \end{cases} \quad (\text{A.1.1})$$

where 'year' is a 12-month period which commences at the start of the krill growth season (1 November), M is the natural mortality rate (effective throughout the year), F_y is the effective annual fishing mortality rate in year y , and a_r is the (knife-edge) age-at-first-capture.

For model 2, which incorporates the extensions described in the main text, these equations become:

$$N_{y+1,a+1} = N_{y,a} e^{-(M+S_a F_y)} \quad 0 \leq a \leq 7 \quad (\text{A.1.2})$$

where S_a is the selectivity-at-age for krill, and describes the relative likelihood of a krill of age a being caught.

These equations assume that the number of krill eight years of age and older is relatively small, and so can be neglected.

Provided that the krill spawning biomass (B^{sp}) is greater than 20% of its median value in the absence of exploitation (K^{sp}), the number of krill recruits at the start of the year ($N_{y,0}$) is given by:

$$N_{y,0} = R e^{\varepsilon_y - \sigma_R^2/2} \quad \varepsilon_y \text{ from } N(0; \sigma_R^2) \quad \text{for } B_y^{sp} \geq 0.2 K^{sp} \quad (\text{A.2})$$

where R is the mean recruitment, and $N(0; \sigma^2)$ is a normal distribution with zero mean and variance σ^2 , i.e. recruitment is independent of spawning biomass provided the spawning biomass is above a certain level. However, below that level, the mean drops proportionately to the spawning biomass:

$$N_{y,0} = \left[\frac{B_y^{sp}}{0.2 K^{sp}} \right] R e^{\varepsilon_y - \sigma_R^2/2} \quad \text{for } B_y^{sp} < 0.2 K^{sp} \quad (\text{A.3})$$

Mass-at-age is obtained from the growth curve form used by Rosenberg *et al.* (1986), together with the assumption that mass is proportional to length cubed. Thus the mass of krill of age a at the start of the 'year' (w_a) is given by:

$$w_a = \alpha [1 - e^{-\beta a}]^3 \quad (\text{A.4})$$

where $\beta = 0.45 \text{ yr}^{-1}$. During the period 1 November to 31 January when krill grow, their mass is:

$$w_a(t) = \alpha [1 - e^{-\beta(a+4t)}]^3 \quad 0 \leq t < 3/12 \quad (\text{A.5})$$

while from February through the balance of the 'year' (growth having ceased):

$$w_a(t) = \alpha \left[1 - e^{-\beta(a+1)} \right]^3 = w_{a+1} \quad 3/12 \leq t < 1 \quad (\text{A.6})$$

In the middle of the growth season their mass-at-age (\bar{w}_a) is thus:

$$\bar{w}_a = \alpha \left[1 - e^{-\beta(a+0.5)} \right]^3 \quad (\text{A.7})$$

For model 1, the spawning biomass of krill (taken to correspond to the start of the 'year') is accordingly given by:

$$B_y^{sp} = \sum_{a=a_m}^7 w_a N_{y,a} \quad (\text{A.8})$$

where a_m is the age-at-maturity for krill.

For model 2 and the alternative fishing strategies (a) to (c), krill is still taken to grow from November to January, so that the 'start' of the year remains 1 November. The spawning biomass of krill is taken to be the average biomass of mature krill over the December to March period (in the presence of an annual catch of krill), and is given by:

$$B_y^{sp} = \sum_{a=0}^7 \int_{1/12}^{5/12} P_a(t) w_a(t) N_{y,a}(t) dt / [5/12 - 1/12] \quad (\text{A.9})$$

where $P_a(t)$ is the proportion of krill of age $(a+t)$ years which are mature (and can be evaluated from equations (3) and (4) of the main text); and $N_{y,a}(t)$ is the number of krill of age a present at time t during the 'year'.

For the alternative fishing strategies (a) to (c), the numbers of krill are given by the following equations. For summer fishing from December to February:

$$N_{y,a}(t) = \begin{cases} N_{y,a} e^{-Mt} & 0 \leq t < 1/12 \\ N_{y,a} e^{-Mt} e^{-4\bar{S}_a F_y(t-1/12)} & 1/12 \leq t < 3/12 \\ N_{y,a} e^{-Mt} e^{-4\bar{S}_a F_y(2/12)} e^{-4S_{a+1} F_y(t-3/12)} & 3/12 \leq t < 4/12 \\ N_{y,a} e^{-Mt} e^{-4F_y(2/12) \bar{S}_a + 1/12 S_{a+1}} & 4/12 \leq t < 1 \end{cases} \quad (\text{A.10a})$$

where \bar{S}_a is assumed for simplicity to be equal to the selectivity corresponding to the average length of krill of age a during the December-January part of the fishing period; this can be evaluated from equation (2) and an appropriate integral of equation (4) of the main text.

Note that during the fishing in February ($3/12 \leq t < 4/12$), krill has stopped growing so that krill length $\ell_a = \ell_{a+1}$ and selectivity = S_{a+1} .

For winter fishing from April to September:

$$N_{y,a}(t) = \begin{cases} N_{y,a} e^{-Mt} & 0 \leq t < 5/12 \\ N_{y,a} e^{-Mt} e^{-2S_{a+1} F_y(t-5/12)} & 5/12 \leq t < 11/12 \\ N_{y,a} e^{-Mt} e^{-2S_{a+1} F_y(6/12)} & 11/12 \leq t < 1 \end{cases} \quad (\text{A.10b})$$

For fishing throughout the year:

$$N_{y,a}(t) = \begin{cases} N_{y,a} e^{-Mt} e^{-\bar{S}'_a F_y t} & 0 \leq t < 3/12 \\ N_{y,a} e^{-Mt} e^{-\bar{S}'_a F_y (3/12)} e^{-S_{a+1} F_y (t-3/12)} & 3/12 \leq t < 1 \end{cases} \quad (\text{A.10c})$$

where \bar{S}'_a is assumed for simplicity to be equal to the selectivity corresponding to the average length of krill of age a during the November-January period of growth.

Note that because F_y is an effective annual fishing mortality, it appears together with the coefficients 4 and 2 for the summer (1/4 year) and winter (1/2 year) fishing seasons of equations (A.10a) and (A.10b) respectively.

Evaluation of equation (A.9) for each of these three cases then yields¹:

$$B_y^{sp} = 3\alpha \sum_{a=0}^7 N_{y,a} \left\{ \begin{aligned} & \left[\bar{P}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+4\mu\beta)/12} \left[1 - e^{-(M+4\bar{S}'_a F_y + 4\mu\beta)(2/12)} \right] \right] / [M + 4\bar{S}'_a F_y + 4\mu\beta] \\ & \times \left[P_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M(3/12) + 4\bar{S}'_a F_y (2/12))} \left[1 - e^{-[M + 4S_{a+1} F_y]/12} \right] \right] / [M + 4S_{a+1} F_y] \\ & + P_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M(4/12) + 4\bar{S}'_a F_y (2/12) + 4S_{a+1} F_y (1/12))} \left[1 - e^{-M/12} \right] \right] / M \end{aligned} \right\} \quad (\text{A.11a})$$

$$B_y^{sp} = 3\alpha \sum_{a=0}^7 N_{y,a} \left\{ \begin{aligned} & \left[\bar{P}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+4\mu\beta)/12} \left[1 - e^{-(M+4\mu\beta)(2/12)} \right] \right] / [M + 4\mu\beta] \\ & \times \left[P_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-M(3/12)} \left[1 - e^{-M(2/12)} \right] \right] / M \end{aligned} \right\} \quad (\text{A.11b})$$

$$B_y^{sp} = 3\alpha \sum_{a=0}^7 N_{y,a} \left\{ \begin{aligned} & \left[\bar{P}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+\bar{S}'_a F_y + 4\mu\beta)/12} \left[1 - e^{-(M+\bar{S}'_a F_y + 4\mu\beta)(2/12)} \right] \right] / [M + \bar{S}'_a F_y + 4\mu\beta] \\ & \times \left[P_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M+\bar{S}'_a F_y)(3/12)} \left[1 - e^{-(M+S_{a+1} F_y)(2/12)} \right] \right] / [M + S_{a+1} F_y] \end{aligned} \right\} \quad (\text{A.11c})$$

where $\gamma(0) = 1$, $\gamma(1) = -3$, $\gamma(2) = 3$ and $\gamma(3) = -1$; and

\bar{P}_a is assumed for simplicity to be equal to the proportion of krill of age a which mature at a length which is the average of that over the December-January part of the spawning season; this can be evaluated from equation (3) and an appropriate integral of equation (4) of the main text.

Spawning biomass B_y^{sp} has to be compared to its median value in the absence of fishing (K^{sp}). As the median of a sum of distributions is not necessarily equal to the sum of their separate medians, K^{sp} is not given exactly by the sum of the median contribution of each age class to the spawning biomass. However, this is a convenient and adequate approximation to make for the purposes of these calculations, so that for model 1 we take:

¹ In the equations that follow continuation is marked by 'x { }' indicating $\sum_{a=0}^7 N_{y,a} \{ \}$

Butterworth *et al.*

$$\begin{aligned}
 K^{sp} &= \sum_{a=a_m}^7 w_a R e^{-\sigma_R^2/2} e^{-Ma} \\
 &= R e^{-\sigma_R^2/2} \alpha \sum_{a=a_m}^7 e^{-Ma} \left[1 - e^{-\beta a}\right]^3
 \end{aligned} \tag{A.12.1}$$

and for model 2:

$$\begin{aligned}
 K^{sp} &= 3\alpha \sum_{a=0}^7 R e^{-\sigma_R^2/2} e^{-Ma} \\
 &\times \left\{ \begin{aligned} &\bar{P}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+4\mu\beta)/12} \left[1 - e^{-(M+4\mu\beta)(2/12)}\right] / [M + 4\mu\beta] \\ &+ P_{a+1} \left[1 - e^{-\beta(a+1)}\right]^3 e^{-M(3/12)} \left[1 - e^{-M(2/12)}\right] / M \end{aligned} \right\}
 \end{aligned} \tag{A.12.2}$$

The survey estimate of biomass is made in the middle of the growth season (i.e. mid-December), and prior to any krill fishing having taken place. For both model 1 and model 2 it is therefore measuring:

$$B_y^{a+} = \sum_{a=a+}^7 \bar{w}_a N_{y,a} e^{-M(1.5/12)} \tag{A.13}$$

where y is the year of the survey (prior to fishing).

The catch by mass of krill from a cohort between times t_1 and t_2 during a year, under actual fishing mortality F , is given by an equation of the form:

$$\int_{t_1}^{t_2} w(t) F N(t) dt$$

For the models considered here, a fishing season from t_1 to t_2 , and an effective annual fishing mortality F_y , the catch by mass in year y is given by:

$$C_y = \sum_{a=0}^7 \int_{t_1}^{t_2} w_a(t) S_a(t) \left[\frac{F_y}{t_2 - t_1} \right] N_{y,a}(t) dt \tag{A.14}$$

For model 1, this simplifies to:

$$C_y = 4 F_y \alpha \sum_{a=a_r}^7 N_{y,a} \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+4F_y+4\mu\beta)(3/12)}\right] / [M + 4F_y + 4\mu\beta] \tag{A.15.1}$$

For model 2, making assumptions as above for selectivity during the growth season, the formulae for the different fishing seasons are:

$$\begin{aligned}
 C_y &= 4F_y \alpha \sum_{a=0}^7 N_{y,a} \\
 &\times \left\{ \begin{aligned} &\bar{S}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+4\mu\beta)/12} \left[1 - e^{-(M+4\bar{S}_a F_y + 4\mu\beta)(2/12)}\right] / [M + 4\bar{S}_a F_y + 4\mu\beta] \\ &+ S_{a+1} \left[1 - e^{-\beta(a+1)}\right]^3 e^{-M(3/12) + 4\bar{S}_a F_y(2/12)} \left[1 - e^{-(M+4S_{a+1} F_y)/12}\right] / [M + 4S_{a+1} F_y] \end{aligned} \right\}
 \end{aligned} \tag{A.15.2a}$$

$$C_y = 2F_y \alpha \sum_{a=0}^7 N_{y,a} S_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-M(5/12)} \left[1 - e^{-(M+2S_{a+1}F_y)(6/12)} \right] / \left[M + 2S_{a+1}F_y \right] \quad (\text{A.15.2b})$$

$$C_y = F_y \alpha \sum_{a=0}^7 N_{y,a} \times \left\{ \begin{aligned} & \bar{S}_a \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+\bar{S}_a'F_y+4\mu\beta)(3/12)} \right] / \left[M + \bar{S}_a'F_y + 4\mu\beta \right] \\ & + S_{a+1} \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M+\bar{S}_a'F_y)(3/12)} \left[1 - e^{-(M+S_{a+1}F_y)(9/12)} \right] / \left[M + S_{a+1}F_y \right] \end{aligned} \right\} \quad (\text{A.15.2c})$$

To effect the refinement of SC-CAMLR (1992) to place a realistic limit on the proportion of the recruited biomass which can be harvested in any year, if the fixed catch specified cannot be attained in year y by an $F_y \leq 1.5 \text{ yr}^{-1}$, then C_y in that year is set equal to the catch achieved for $F_y = 1.5$.

SC-CAMLR (1992) requested calculation of production:biomass (P/B) ratios. The production \tilde{P}_y achieved by the resource in year y is the sum of the increase in biomass, the catch made by the fishery, and the loss to natural predators (reflected through natural mortality M), i.e.:

$$\tilde{P}_y = B_{y+1} - B_y + C_y + P_y^r \quad (\text{A.16})$$

where B_y is the biomass at the 'start' of year y , and P_y^r is the biomass consumed by natural predators during year y .

The biomass is given by the equation:

$$B_y = \sum_{a=0}^7 w_a N_{y,a} \quad (\text{A.17})$$

while predation loss over the whole year is evaluated from an expression of the form:

$$\int_0^1 w(t) M N(t) dt$$

which here takes the more specific form:

$$P_y^r = \sum_{a=0}^7 \int_0^1 w_a(t) M N_{y,a}(t) dt + w_8 N_{y+1,8} \quad (\text{A.18})$$

The reason for the final term in equation (A.18) is that the neglect of krill of eight years of age and older - implicit in equations (A.1.1) and (A.1.2) - must be accounted for here by 'pretending' that all these krill are lost to natural mortality on the verge of their eighth birthday.

Evaluation of the integrals in (A.18) for the various models and fishing seasons yields results similar to those of equations (A.15), except that extra terms have to be added for cases when fishing does not extend over the whole year. Thus, for model 1:

$$\begin{aligned}
P_y^r &= M \alpha \sum_{a=0}^7 N_{y,a} \\
&\times \left\{ \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+4\tilde{S}_a F_y + 4\mu\beta)(3/12)} \right] / [M + 4\tilde{S}_a F_y + 4\mu\beta] \right. \\
&\quad \left. + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M+4\tilde{S}_a F_y)(3/12)} \left[1 - e^{-M(9/12)} \right] / M \right\} \\
&+ w_8 N_{y+1,8}
\end{aligned} \tag{A.19.1}$$

$$\text{where } \tilde{S}_a = \begin{cases} 0 & \text{for } a < a_r \\ 1 & \text{for } a \geq a_r \end{cases}$$

For model 2:

$$\begin{aligned}
P_y^r &= M \alpha \sum_{a=0}^7 N_{y,a} \\
&\times \left\{ \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+4\mu\beta)/12} \right] / [M + 4\mu\beta] \right. \\
&\quad + \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} e^{-(M+4\mu\beta)/12} \left[1 - e^{-(M+4\tilde{S}_a F_y + 4\mu\beta)(2/12)} \right] / [M + 4\tilde{S}_a F_y + 4\mu\beta] \\
&\quad + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M(3/12) + 4\tilde{S}_a F_y(2/12))} \left[1 - e^{-(M+4S_{a+1} F_y)/12} \right] / [M + 4S_{a+1} F_y] \\
&\quad \left. + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M(4/12) + 4\tilde{S}_a F_y(2/12) + 4S_{a+1} F_y(1/12))} \left[1 - e^{-M(8/12)} \right] / M \right\} \\
&+ w_8 N_{y+1,8}
\end{aligned} \tag{A.19.2a}$$

$$\begin{aligned}
P_y^r &= M \alpha \sum_{a=0}^7 N_{y,a} \\
&\times \left\{ \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+4\mu\beta)3/12} \right] / [M + 4\mu\beta] \right. \\
&\quad + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-M(3/12)} \left[1 - e^{-M(2/12)} \right] / M \\
&\quad + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-M(5/12)} \left[1 - e^{-(M+2S_{a+1} F_y)(6/12)} \right] / [M + 2S_{a+1} F_y] \\
&\quad \left. + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M(11/12) + 2S_{a+1} F_y(6/12))} \left[1 - e^{-M/12} \right] / M \right\} \\
&+ w_8 N_{y+1,8}
\end{aligned} \tag{A.19.2b}$$

$$\begin{aligned}
P_y^r &= M \alpha \sum_{a=0}^7 N_{y,a} \\
&\times \left\{ \sum_{\mu=0}^3 \gamma(\mu) e^{-\mu\beta a} \left[1 - e^{-(M+\tilde{S}_a' F_y + 4\mu\beta)(3/12)} \right] / [M + \tilde{S}_a' F_y + 4\mu\beta] \right. \\
&\quad \left. + \left[1 - e^{-\beta(a+1)} \right]^3 e^{-(M+\tilde{S}_a' F_y)(3/12)} \left[1 - e^{-(M+S_{a+1} F_y)(9/12)} \right] / [M + S_{a+1} F_y] \right\} \\
&+ w_8 N_{y+1,8}
\end{aligned} \tag{A.19.2c}$$

2. ALGORITHM FOR THE COMPUTATIONS

The results required for model 2 are computed using the following algorithm. The algorithm for model 1 is similar, and is provided in detail in Butterworth *et al.* (1992), except that subsequent modifications, as reflected in steps (viii) to (xii) described below, have now been incorporated.

- (i) Choose a value for γ (δ), which is used in conjunction with the following pre-specified values of biological and other parameters:

$$\begin{aligned}\beta &= 0.45 \text{ yr}^{-1} \\ \alpha &= 1 \\ R &= 1 \end{aligned} \left. \vphantom{\begin{aligned}\beta &= 0.45 \text{ yr}^{-1} \\ \alpha &= 1 \\ R &= 1 \end{aligned}} \right\} \text{ since results are required for biomass ratios only}$$

$$\begin{aligned}\sigma_s &= 0.3 \\ a+ &= 1+ \text{ yr}\end{aligned}$$

- (ii) Choose values for the following biological parameters (new choices are made for each 31-year simulation):

$$\begin{aligned}M &\text{ from } U[0.4, 1.0] \text{ yr}^{-1} \quad \{\text{natural mortality}\} \\ \ell_r^{50} &\text{ from } U[38, 42] \text{ mm} \quad \{\text{length at 50\% selectivity}\} \\ \ell_m^{50} &\text{ from } U[34, 40] \text{ mm} \quad \{\text{length at 50\% maturity}\} \\ \sigma_R &\text{ from } U[0.4, 0.6] \quad \{\text{recruitment variability}\} \\ f &= 1, \text{ or} \\ f &\text{ from } U[1, 4] \quad \{\text{if incomplete survey coverage is taken into account}\}\end{aligned}$$

- (iii) Set $\lambda = \gamma/M$ [or $\lambda = \delta/M$]

- (iv) For 'year' $y = 1$, set up deterministic equilibrium age-structure for $F = 0$, i.e.

$$N_{1,a} = R e^{-\sigma_R^2/2} e^{-Ma} \quad a = 0, 1, \dots, 7 \quad (\text{A.20})$$

- (v) Project the numbers-at-age forward in the absence of exploitation [$F_y = 0$] using equation (A.1.2), with stochastic recruitment given by equation (A.2), for 'years' $y = 2$ to 11. Thus, by 'year' $y = 10$, all memory of the initial deterministic age-structure has been lost. To avoid any possible confounding of interpretation of results through the stock-recruit relationship of equation (A.3), this relationship comes into effect only after the first ten years, i.e. it is considered first when computing $N_{11,0}$.

- (vi) In 'year' $y = 10$ (the year before fishing commences), 'measure' the $a+$ biomass in mid-season (i.e., the middle of the growth season: mid-December, to yield a result B_{survey} where:

$$B^* = \sum_{a=a+}^7 \bar{w}_a N_{10,a} e^{-M(1.5/12)} e^{-\eta - \sigma_s^2/2} \quad (\text{A.21})$$

$$B_{\text{survey}} = B^* / f$$

where η from $N(0; \sigma_s^2)$ is the lognormal variation associated with the biomass survey estimate.

- (vii) Fix a constant catch $C = \gamma B_{\text{survey}}$ for complete survey coverage ($f = 1$) (otherwise $C = \delta B_{\text{survey}} = \lambda M f B_{\text{survey}}$).

- (viii) For year $y = 11$, find the effective annual fishing mortality F_y which satisfies equation (A.15.2) for the catch computed in step (vii). Care must be taken in this process because the combined effects of biomass gains, arising from the growth of individual krill, and biomass losses to natural mortality, may render C_y a non-monotonic function of F_y (cf. standard yield-per-recruit computations). However, this does not occur for the winter fishing season during which there is no growth of individual krill. Computations are effected in a manner that provides the lowest value of F_y satisfying equation (A.15.2) in the event of multiple solutions. If

no solution is possible for $F_y \leq 1.5$, then F_y is set to 1.5 yr^{-1} and the constant catch (C) of step (vii) is replaced (for this year only) by the value of C_y corresponding to this maximal fishing mortality.

- (xi) Calculate the spawning biomass B_y^{sp} for year 11 by means of equation (A.11), and follow this by the calculation of the ratio of this biomass to the corresponding median level in the absence of fishing: $D_y = B_y^{sp} / K^{sp}$ for $y = 11$. [Note that this ratio is impacted by fishing in year $y = 11$ for a summer fishing season or fishing throughout the year, as the spawning and fishing periods overlap in those cases. However, for a winter fishing season (or for model 1), year $y = 12$ is the first year that spawning biomass is influenced by the effects of fishing.]
- (x) Project the age-structure of the population forward to year $y = 12$ using the value of F_y calculated in step (viii) and equation (A.1.2), together with equation (A.2) or (A.3) as appropriate. Losses to predation (P_y') for $y = 11$ can now be calculated by means of equation (A.19), and thence production \tilde{P}_y for year 11 through equation (A.16).
- (xi) Repeat steps (viii) to (x) for $y = 12$ to 31, thereby obtaining values of D_y for $y = 12$ to 31.
- (xii) Ascertain whether any elements of the set $\{D_y: y = 1, \dots, 31\}$ are less than the various 'critical' values considered ($D_{crit} = 0.1, 0.2, \dots, 0.6$). [Note that this procedure is identical to that of Butterworth *et al.* (1992) for model 1 and for model 2 with a winter fishing season, as it considers a series of exactly 20 spawning biomasses affected by fishing in these two cases, for reasons explained in step (ix). This procedure has been maintained for consistency in the other two cases of summer and whole-season fishing under model 2, even though this means that 21 (i.e., one extra) spawning biomass values impacted by fishing are considered.]
- (xiii) Repeat steps (ii) to (xii) a large number of times (1 000 simulations were found to provide adequate precision for this study), to estimate the probability (P) that a value of D_y less than specified critical values occurs during the 20-year harvesting period. Also obtain the median, and 5% and 95% distribution points, of $D(31) = B_{fin} / K^{sp} = B_{31}^{sp} / K^{sp}$ from the set of values obtained for B_{fin} / K^{sp} for each 31-year simulation, as well as the requisite average of the annual P/B ratio.
- (xiv) Repeat steps (i) to (xiii) for different values of γ (or δ).

Note that to ensure a smooth relationship between γ (or δ) and probability (P), the same set of random numbers was used for each γ (or δ) value considered. Because of the approximation of equation (A.12) and Monte Carlo variation, the median value obtained for B_{fin} / K^{sp} when γ (or δ) = 0 differed marginally from 1. Results shown in the figures were rescaled so that plots of median B_{fin} / K^{sp} intercepted the axis at 1 exactly.

The curves for probability were further smoothed using seven-point quadratic smoothing. Values in Tables 1 and 2 were obtained by interpolation.